1. Introduction

Heavy-ion collision experiments provide us a unique opportunity to study the exciting phenomena happening in nucleus–nucleus interactions at relativistic high energies. Among these phenomena, limiting fragmentation retains its own unique significance from the very beginning since it was first proposed by Benecke et al. in 1969. Since then, a special attention has been paid by physics community associated with various heavy-ion experiments. According to which, in a high energy collision, in the frame where a target or a projectile is at rest, some of the outgoing particles approach limiting distributions. That is to say, the distribution exhibits a negligible change at finite bombarding energy for large $S$ (the square of center-of-mass energy) such as the pseudo-rapidity density as a function of $\eta - Y_{\text{beam}}$ of the outgoing particles near beam or target rapidity for various energies appears to approach a fixed curve in the fragmentation region. This hypothesis is based on the geometrical picture of scattering as considered by Yang and coworkers. According to their assumption, in the laboratory frame the projectile nuclei undergoes a Lorentz contraction in the form of a thin disk (or pancake) in the collision with the target nuclei which is further and further compressed with the increasing energy. However, the momentum and quantum number transfer process between the projectile and target does not appreciably change when the projectile is further and further compressed. This behavior leads the outgoing particles to retain the limiting fragmentation behavior in high energy collisions. Because of the large separation between the projectile and target fragments, limiting fragmentation admits no correlation between projectile and target fragments. The hypothesis of limiting fragmentation, an asymptotic property of the nucleus–nucleus collisions at relativistic high energies, has been observed experimentally for a variety of collision processes such as hadron–hadron, hadron–nucleus, and nucleus–nucleus interactions for produced charged hadrons at different energies and also for photons at forward pseudo-rapidity. Apart from the multitude of produced charged hadrons, the light, intermediate and heavy fragments (which are mainly the decay products of the projectile spectator and found near the beam rapidity region) are also emitted in a substantial amount in these heavy-ion collision experiments. The main sources of production of these fragments are nucleons and nucleon clusters formed in these nuclear collisions. Out of these nuclear fragments, projectile helium fragments with charge $Z = 2$, are most abundant in the projectile fragmentation region and are of great importance in studying the fragmentation process of the projectile nuclei. In this perspective, the study of limiting fragmentation phenomenon for these projectile helium fragments needs some more attention to be paid in order to understand the underlying physics of the fragmentation mechanism involved in such nuclear collisions at relativistic high energies. The projectile helium fragments with charge $Z = 2$, have been studied earlier in nuclear emulsion experiments from the point of view of limiting fragmentation by Heckman et al. and by Bhanja et al. at 2.1 A GeV energy. In this sequence, some other authors confirmed this hypothesis experimentally for projectile helium fragments in nucleus–nucleus interactions in later years. Such a quantitative study of limiting fragmentation hypothesis on the experimental basis for the projectile helium fragments in nucleus–nucleus interactions at 14.6 A GeV has not been performed so far. Recently, we have performed an extensive study to investigate some of the general characteristics of these projectile helium fragments emitted in nucleus–nucleus interactions for the same energy. In the present paper, we discuss our detailed analysis from the point of view of testing the limiting fragmentation behavior in case of projectile helium fragments produced in nucleus–nucleus interactions at 14.6 A GeV energy, which will also throw some more light on the significant role of the fragmentation region in relativistic nuclear collisions. For this purpose, we discuss quantitatively projected angular distribution and average emission angle in individual helium reaction channels with different emulsion targets. We have also shown the energy independent behavior of limiting fragmentation phenomenon in the fragmentation region for projectile helium fragments in nucleus–nucleus interactions.
2. Experimental Details

A stack of Fuji emulsion pellicles exposed horizontally to a $^{28}\text{Si}$ beam at 14.6 A GeV at BNL AGS, was employed to collect the data sample used in the present analysis. To locate the minimum-bias $^{28}\text{Si}$-emulsion interaction events, conventional along-the-track scanning technique was used. The emulsion pellicles were scanned by using an Olympus BH2 microscope with a 100× oil immersion objective under a total magnification of 2250. Each primary beam track in an emulsion pellicle was carefully followed up to a distance of 4 cm from the entrance edge. Events produced very close to top or bottom surface of the emulsion pellicle up to 20 µm were not taken into account for the current investigation. A total 855 inelastic events fulfilled our selection criteria and this data set is finally used in the present investigation. Charged secondary particles emitted in each interaction were divided according to their ionization, range and velocity into black (b), grey (g), shower (s), and projectile fragments (PFs) having charge $Z \geq 2$. Black particles are slow velocity particles with $\beta < 0.3$ having range less than 3 mm in emulsion and ionization $g > 6g_{\text{min}}$, where $g_{\text{min}}$ is the grain density of singly charged particle moving with velocity close to initial beam velocity. These are low energy, multiply charged fragments and are mainly evaporated particles from the target nuclei. Grey particles have a range greater than 3 mm and ionization $1.4g_{\text{min}} < g \leq 6g_{\text{min}}$. These particles are mainly knocked out protons from the target nucleus. Both black and grey tracks are target fragments emanating from emulsion target in its excited state. Shower particles are having ionization $g \leq 1.4g_{\text{min}}$ and velocity $\beta > 0.7$. Shower particles are mainly relativistic pions, with small fraction of K-meson, fast protons and anti-protons. Projectile fragments (PFs) with $Z \geq 2$ are having $g \geq 4g_{\text{min}}$, emitted in a narrow forward cone. The multiplicities of black, grey, shower and projectile fragments are denoted by $n_{\text{b}}$, $n_{\text{g}}$, $n_{\text{s}}$, and $n_{\text{p}}$ respectively. Black and grey particles collectively are called heavily (h) ionizing particles such that $n_{\text{h}} = n_{\text{b}} + n_{\text{g}}$ and they can be emitted by an excited target nucleus in any direction with respect to the primary beam track. Primary fragments with $Z = 2$ are denoted as projectile helium fragments, which are solely identified from their grain density (about $4g_{\text{min}}$). Projectile helium fragments, in general, traverse in the very forward cone in the direction of primary $^{28}\text{Si}$ beam. This cone angle can be obtained from the primary beam energy of 14.6 A GeV in conjunction with the Fermi momentum.

3. Target Identification and Helium Reaction Channels

Nuclear emulsion is composed of different targets H, C, N, O, Ag, Br, and I nuclei. A clear identification of these different targets in nuclear emulsion is not so straightforward. Statistically, identification of collision events with different target nuclei in nuclear emulsion is performed on the basis of the multiplicity of heavy particles ($n_{\text{h}}$), which is a function of the size of the target and also is a characteristics of the impact parameter. On this basis, different collision events in nuclear emulsion is characterized as follows:

(1) H-events: The events which fall under the criteria $n_{\text{h}} \leq 1$ are collision events with hydrogen nuclei in the nuclear emulsion. These are mainly peripheral collision events.

(2) CNO-events: The events which fall under the criteria $2 \leq n_{\text{h}} \leq 8$ are collision events with light emulsion target nuclei C, N, and O with average mass number 14. These collision events may also contain some peripheral collision events with Ag and Br nuclei.

(3) AgBr-events: The events which fall under the criteria $n_{\text{h}} \geq 8$ are collision events with heavy target nuclei (Ag and Br nuclei) in the nuclear emulsion. These events correspond to non-peripheral collision events with Ag and Br nuclei in nuclear emulsion.

In Fig. 1, we have shown the variation of mean multiplicity of projectile helium ($Z = 2$) fragments as a function of the mean multiplicity of heavily ionizing particles ($n_{\text{h}}$). A strong correlation exists between the production of heavily ionizing particles (i.e., target fragments) and the projectile helium fragments. We can clearly observe that the mean multiplicity of projectile helium fragments decreases linearly with the increase in the mean multiplicity of heavily ionizing particles (target fragments). The events with $n_{\text{h}} \leq 1$ are accompanied by the large multiplicity of projectile helium fragments due to the extreme peripheral nature of the collision event. The events with $n_{\text{h}} \geq 28$ are having lowest multiplicity of the projectile helium fragments due to the central collision with AgBr nuclei. Thus, we can see that with increasing degree of centrality, the emission of projectile helium fragments decreases.

We have employed a data set of 855 minimum-bias inelastic events for the present scholarly article. Out of the total 855 events, the inelastic events with an emission of one or more projectile helium fragments are further sub-divided into different types of helium reaction channels depending on the multiplicity of projectile helium fragments emitted in an individual interaction event, the emission of helium fragments increases as we go form the 1 × He to $n \times \text{He}$ ($n = 1, 2, \ldots, 6$) channel. These different helium reaction channels provide us an alternative way to investigate the variation of various limiting characteristics of projectile fragments with the variation of degree of projectile breakup.

In a sub-sample of events associated with one or more projectile helium fragments, 53% events correspond to 1He
channel with or without emission of projectile fragments with charge $Z \geq 3$, 29% events correspond to 2He channel, 10% events correspond to 3He channel and 7% events correspond to 4He channel. There are very few events with 5He and 6He channels. Therefore, we have mixed these two channels with 4He channel and thus collectively categorized them as 4–6He channel (8% events). The interaction events corresponding to this channel are extremely peripheral in nature.

4. Projected Angular Distribution of Projectile Helium Fragments

In Fig. 2, limiting fragmentation is experimentally investigated in terms of the measured projected angular distribution ($\theta_p$) of the projectile helium fragments in individual helium reaction channels. Here, different reaction channels originate from inelastic nuclear collisions of varying impact parameters starting with the interactions at very small impact parameter to the interactions at quite large impact parameter. The projected angular distribution of projectile helium fragments in each individual reaction channel is fitted with a Gaussian function of the form $N(\theta_p) = A \exp(-\theta_p^2/2\sigma^2)$. Projected angular distribution for each individual helium reaction channel is displayed with its respective $\chi^2$ per degree of freedom and the standard deviation $\sigma$. From Fig. 2, we can observe that standard deviation of the projected angular distributions for individual helium channels is more or less comparable to each other qualitatively within statistical error. Thus, on the basis of the comparable values of $\sigma$ of projected angular distributions in individual helium channels, we can observe that the emission characteristics of projectile helium fragments are independent of the degree of excitation of the projectile. As the degree of excitation of projectile is a quantitative measure of the degree of centrality of the collision events, we can say that the emission properties of projectile helium fragments do not depend upon the degree of centrality of the collision events. Hence, we can infer that the hypothesis of limiting fragmentation is evidently independent of the degree of centrality of the collision events for projectile helium fragments. In Fig. 3, limiting fragmentation behavior is tested on the basis of measured projected angular ($\theta_p$) distribution for projectile helium fragments as a function of different emulsion target groups displayed along with their standard deviation $\sigma$. The
projected angular ($\theta_p$) distributions for projectile helium fragments with different emulsion target groups have been fitted with a Gaussian function of the form mentioned earlier. From Fig. 3, we can see that the standard deviation width of the projected angular distributions for H-events ($n_h = 0, 1$), CNO-events ($2 \leq n_h \leq 8$), and for minimum-bias events ($n_h \geq 0$) are comparable to each other quantitatively within statistical error whereas the value of width for projectile helium fragments shows a little higher value for AgBr-events which corresponds to the subset of events with higher degree of target excitation. This little deviation in the value of width of projected angular distribution in AgBr-events for projectile helium fragments has been observed previously in $^{12}$C-emulsion interactions at 4.5 A GeV/$c$.\textsuperscript{23} Thus, it is evident that the value of width of the projected angular distributions for the projectile helium fragments are qualitatively comparable to each other within statistical errors. These observations demonstrate that fragmentation properties of the projectile helium fragments are almost independent of the mass of the different emulsion targets, which is compatible with the hypothesis of limiting fragmentation.

5. Average Emission Angle in Individual Helium Channels with Different Target Groups

In this section, we have depicted a tabular representation of the value of average emission angle ($\langle \theta_{He} \rangle$) in each individual helium reaction channels with three different emulsion-target groups (i.e., H-events, CNO-events, and AgBr-events). The emission angle of projectile helium fragments is the angle measured from the direction of the incident projectile nuclei. In the present work, it is calculated by using the projected angle ($\theta_p$) in $XY$-plane and dip angle measured in the $Z$-direction (i.e., vertical direction) of the emitted particle in each event. From Table I, if we examine the dependence of the value of average emission angle ($\langle \theta_{He} \rangle$) of projectile helium fragments as a function of different target groups, we can clearly see that in each individual helium channel, the average value of emission angle remains more or less the same quantitatively within statistical errors except in AgBr-events. For AgBr-events, the value of average emission angle is slightly larger in $^{1}$He and $^{2}$He channels in comparison to the value of average emission angle in $^{3}$He, $^{4}$He, and $^{4-6}$He channels.

Fig. 3. Observed projected angular distribution ($\theta_p$) for projectile helium ($Z = 2$) fragments with different groups of target nuclei. Solid curve is a Gaussian fit to the data points: (a) H-events, (b) CNO-events, (c) AgBr-events, and (d) minimum-bias events.

<table>
<thead>
<tr>
<th>Target Group</th>
<th>$\chi^2$/ndf</th>
<th>$0.24 \pm 0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-events</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>CNO-events</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>AgBr-events</td>
<td>1.2</td>
<td>0.27 \pm 0.02</td>
</tr>
<tr>
<td>Minimum-bias</td>
<td>1.6</td>
<td>0.24 \pm 0.10</td>
</tr>
</tbody>
</table>

Table I. Average Emission Angle in Individual Helium Channels with Different Target Groups.
slightly enhanced value of average emission angle for AgBr-events may be attributed to the fact that such forward moving helium fragments have been imparted a large transverse momentum. This observation gives a clear indication that the value of average emission angle ($\langle \theta_{He} \rangle$) of projectile helium fragments is qualitatively independent of the mass of the all distinct target groups. Similarly, if we analyze the value of average emission angle ($\langle \theta_{He} \rangle$) of projectile helium fragments in individual helium channels (i.e., with the degree of excitation of projectile), we can see that the value of $\langle \theta_{He} \rangle$ remains almost constant quantitatively within statistical errors from very low degree of excitation of projectile (i.e., extreme peripheral collision events) up to higher degree of excitation of projectile (i.e., central collision events) with different target groups and also in minimum-bias events. Thus, on the basis of quantitative analysis of the emission angle of projectile helium fragments with various target groups in individual helium channel, we can say that the value of average emission angle ($\langle \theta_{He} \rangle$) of projectile helium fragments is independent of the mass of different emulsion target groups and the degree of excitation of projectile nuclei (i.e., centrality of the collision events). This observation gives ample support to the hypothesis of limiting fragmentation in nucleus–nucleus interactions at 14.6 A GeV.

6. Energy Dependence of Limiting Fragmentation Behavior for Projectile Helium Fragments

In this section, we have presented the results on the longitudinal scaling of projectile helium fragments produced in nucleus–nucleus interactions at 14.6 A GeV. In order to investigate the ansatz of limiting fragmentation, we present the pseudo-rapidity distribution of projectile helium fragments in rest frame of the projectile beam by using the beam rapidity value $Y_{beam}$ mentioned in ref. 27. The pseudo-rapidity variable of the emitted particle in nucleus–nucleus interactions is defined in the following manner:

$$\eta = - \ln \left[ \tan \left( \frac{\theta}{2} \right) \right],$$

where $\theta$ is the emission angle of the emitted particles from the beam axis.

In Fig. 4, we have shown the normalized pseudo-rapidity distribution of projectile helium fragments in order to study the energy dependence of limiting fragmentation. For this purpose, we have made a comparison of our data with the published data in similar experiment at different energy (3.7 A GeV28) in minimum-bias inelastic events. Both the distributions are fitted with a Gaussian function. The value of the standard deviation width of the distribution of projectile helium fragments at 14.6 and 3.7 A GeV obtained from the Gaussian fits are 2.32 and 2.18, respectively. The pseudo-rapidity distributions of projectile helium fragments have peak value around 0.56 and 0.64 at 14.6 and 3.7 A GeV respectively. This difference in the values of the heights and the standard deviation widths of both the distributions may be due to the difference in the energy of the projectile nuclei. A similar decrease in the height of the normalized pseudo-rapidity distribution with increase in energy of the projectile nuclei can be clearly seen as observed in refs. 29 and 30. Furthermore investigations are required in order to give a more detailed explanation of the difference in the normalized pseudo-rapidity distributions of projectile helium fragments due to the effect of the energy of the projectile nuclei. From Fig. 4, we distinctly witness the evidence for limiting fragmentation in projectile rest frame, where the two distributions overlap on a common limiting curve for the values above $\eta \approx 2.40$, covering more than half of the available range of $\eta - Y_{beam}$ over which projectile helium fragments are produced. Furthermore, we can observe that both the distributions start approximately for the same $\eta$ value and tails of both the distributions end at an almost identical $\eta$ value in projectile rest frame. Thus, the projectile helium fragments are found to follow an energy independent limiting fragmentation behavior.

7. Summary and Conclusions

From the extensive analysis of projectile helium ($Z = 2$) fragments emerged in $^{28}$Si induced emulsion interactions at 14.6 A GeV, we may conclude our research work with the interesting observations that projected angular distributions of projectile helium fragments in individual helium reaction channel and with different emulsion targets exhibit a limiting fragmentation behavior which holds good for inelastic nucleus–nucleus interactions at 14.6 A GeV. An analysis of average emission angle of projectile helium fragments also shows the limiting fragmentation behavior, which is found to be independent of individual helium reaction channels (i.e., degree of projectile breakup in different collision events) and of the mass of different emulsion targets. Furthermore, a comparative study of pseudo-rapidity distributions of projectile helium fragments at two different energies is performed, which clearly demonstrates an energy independent behavior of limiting fragmentation in the projectile fragmentation region.
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