Study of Projectile Helium \((Z = 2)\) Fragments Produced in Nucleus–Nucleus Interactions at 14.6 A GeV

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An analysis of projectile helium \((Z = 2)\) fragments emitted from 855 minimum-bias events of \(^{28}\)Si projectiles in nuclear emulsion at 14.6 A GeV has been performed. An event-by-event examination has been made from the minimum-bias events in order to see the dependence of shower and target fragments multiplicities in individual helium reaction channels. The validity of KNO scaling for the projectile helium \((Z = 2)\) fragments has also been studied. The angular and derived pseudorapidity and momentum distributions of projectile helium \((Z = 2)\) fragments are found to be characteristically Gaussian shaped with some width \(\sigma\).

KEYWORDS: \(^{28}\)Si at 14.6 A GeV, projectile helium fragments, multiplicities, KNO scaling, space angle, pseudorapidity distribution, transverse momentum distribution

1. Introduction

Heavy ion collisions at relativistic energies cause the nuclear matter to undergo the extreme conditions of high energy and the baryon density. Under such extreme conditions new forms of matter such as the quark-gluon plasma (QGP)\(^1\)–\(^4\) can be produced in the laboratory which might be formed in central collisions. However, a number of other interesting aspects in heavy ion physics can be studied in the peripheral collisions.\(^5\)–\(^7\) In such heavy ion collisions, nucleons of the spectator parts of projectile nucleus do not interact strongly and they form the residual nuclear systems, projectile helium fragments are part of these nuclear systems. As these relativistic projectile helium fragments are the major part of the projectile fragments and also they carry the rapidity of the projectile beam so they can provide us the necessary information about the fragmentation mechanism of heavy ion nucleus–nucleus collisions. Therefore, our main motivation is to investigate the various important features of projectile helium \((Z = 2)\) fragments and to develop a better understanding about the production mechanism of these particles. In this paper, we present the results on the multiplicity of shower and heavily ionizing particles on the basis of individual projectile helium \((Z = 2)\) reaction channels. We have also analyzed the results on space angle distribution, pseudo-rapidity distribution and transverse momentum distribution of projectile helium \((Z = 2)\) fragments produced in \(^{28}\)Si–emulsion interactions at our energy. An attempt has also been made to check the projectile helium \((Z = 2)\) fragments multiplicity scaling.

2. Experimental Details

A stack of Fuji emulsion pellicles exposed horizontally with a \(^{28}\)Si beam at 14.6 A GeV at BNL AGS, was used to obtain the present data for our analysis. To locate the minimum-bias \(^{28}\)Si–emulsion interaction events, conventional along-the-track scanning technique was used. The emulsion pellicles were scanned by using an Olympus BH2 microscope with a 100 \(\times\) oil immersion objective under a total magnification of 2250. Each primary beam track in emulsion pellicles was carefully followed up to a distance of 4 cm from the entrance edge. Events produced very close to top or bottom surface of the emulsion up to 20 \(\mu\)m were not taken into account for the investigation. Total 855 inelastic events were taken into account for our investigation purposes. Charged secondary particles emitted in each interactions were divided according to their ionization, range and velocity into black \((b)\), grey \((g)\), shower \((s)\) and projectile fragments \((PF)\) having charge \(Z \geq 2\). Black particles are slow velocity particles with \(\beta < 0.3\) having range less than 3 mm in emulsion and ionization \(g > 6g_{\text{min}}\) where \(g_{\text{min}}\) is the grain density of singly charged particle moving with velocity close to initial beam velocity. These are low energy, multiply charged fragments and are mainly evaporated particles from the target nuclei. Grey particles have a range greater than 3 mm and ionization 1.4\(g_{\text{min}} < g \leq 6g_{\text{min}}\). These particles are mainly knocked out proton from the target nuclei. Black and grey both the particles are target fragments. Shower particles are having ionization \(g \leq 1.4g_{\text{min}}\) and velocity \(\beta > 0.7\). Shower particles are mainly relativistic pions, with small fraction of K-meson, fast protons and anti-protons. Projectile fragments \((PF)\) with \(Z \geq 2\) are having \(g \geq 4g_{\text{min}}\), emitted in a narrow forward cone. The multiplicities of black, grey, shower and projectile fragments are denoted by \(n_b, n_g, n_s,\) and \(n_p\) respectively. Black and grey particles collectively are called heavily \((h)\)-ionizing particles such that \(n_b = n_0 + n_g\). For \(Z = 2\) case, we denote the PF\(_{h}\) as a projectile helium fragments. Projectile helium fragments are identified from their grain density solely, which is about 4\(g_{\text{min}}\).

3. Multiplicity Distribution of Projectile Helium \((Z = 2)\) Fragments

The multiplicity distribution of the projectile helium fragments produced in \(^{28}\)Si–emulsion interactions at 14.6 A GeV energy, has been presented here. In Fig. 1, we have shown the normalized multiplicity distribution of the projectile helium fragments at 14.6 A GeV along with a comparison with the data for the same projectile beam at 3.7 A GeV\(^8\) and the data for different projectile \(^{32}\)S beam at 200 A GeV energy.\(^9\) From Fig. 1, we observe a decreasing trend of the distribution at 14.6 A GeV energy with a tail extending up to \(n_p = 6\). The distribution at 3.7 A GeV\(^8\) also follow the same decreasing trend where as the distribution
for $^{32}$S at 200 A GeV\(^{9}\) shows a little difference with higher peak at $n_n = 0$. It is evident that $n_n = 0$ events are most frequent for all projectiles (between 42–45% events in each case), this class of events ($n_n = 0$) does not have any $Z = 2$ PF, but may have the PFs of charges $Z = 1$ and $Z \geq 3$. All three sets of observations clearly exhibit energy independence indicating to agree well in most of helium fragments channels associated with it. In other experiments\(^{10}\) for $^{16}$O–emulsion interactions at 14.6 A GeV, the following frequency distribution for different helium channel events was obtained: 60.40 $\pm$ 2.64 for events with $n_n = 0$, 19.30 $\pm$ 1.70 for events with $n_n = 1$, 13.40 $\pm$ 1.50 for events with $n_n = 2$, 6.70 $\pm$ 1.10 for events with $n_n = 3$ and 0.20 $\pm$ 0.20 for events with $n_n = 4$, respectively. As the mass as well as the energy of projectile (from $^{28}$Si to $^{32}$S) increases, probability for $n_n = 0$ events increases due to the higher probability for central collisions. Out of 855 minimum bias events, 42% event are observed having no projectile helium fragments in case of $^{28}$Si–emulsion interactions at 14.6 A GeV which is same as in case of 3.7 A GeV\(^{8}\) where as in the case of $^{32}$S at 200 A GeV energy,\(^{9}\) 45% events are observed having no projectile helium fragments. These events correspond mostly to the central collision events in which no helium fragments emitted from projectile nuclei is seen. The events corresponding to $n_n = 5$ and 6 are very less, which is mainly due to the extreme peripheral nature of the collision events. The unbiased sample of 855 inelastic events have the average number of projectile helium fragments emitted per interaction is ($n_n$) $= 1.01 \pm 0.03$. After excluding the events having no helium fragments emitted from the projectile nuclei, we have the average number of projectile helium fragments per interaction: ($n_n$) $= 1.74 \pm 0.08$ for 496 events as shown in Table I along with a comparison with $^{28}$Si data at 3.7 A GeV\(^{8}\) and $^{32}$S at 200 A GeV\(^{9}\) respectively.

In recent years, the validity of Koba–Nielson–Olesen (KNO) scaling\(^{11}\) of the projectile helium fragments produced in nucleus–nucleus interactions at high energies has been reported in many experiments.\(^{8,12,14}\) By assuming the Feynman scaling of the inclusive particle production cross section, Koba et al.\(^{11}\) predicted that the normalized multiplicity distribution of produced charged particles should become independent at asymptotic energies. According to the KNO scaling, the multiplicity distribution is expressed as a function of $z$ (= n/(n)) in the following form:

$$
\Psi(z) = \langle n \rangle P(n) = \langle n \rangle \frac{\sigma_n}{\sigma_{tot}},
$$

(1)

where $\langle n \rangle$ is the mean multiplicity. $\sigma_n$ is the partial cross section for the n particle production and $\sigma_{tot}$ is the total inelastic cross section respectively. The multiplicity of projectile helium fragments can also be well represented by the universal function of the form,

$$
\Psi(z) = A z \exp(-B z),
$$

(2)

where A and B are constants. In Fig. 2, we have plotted $\Psi(z)$ as a function of $z$ for projectile helium ($Z = 2$) fragments in $^{28}$Si–emulsion interactions at 14.6 A GeV. In order to find a possible similar features in the case of PF, from the different systems, we have taken the data of $^{24}$Mg–emulsion interactions at 3.7 A GeV.\(^{15}\) From Fig. 2, one may observe that qualitatively the data points for $^{28}$Si and $^{24}$Mg beam lie on a common curve consistent with scaling behavior for projectile helium fragments. For our experiment, the value of coefficients A and B are 6.55 $\pm$ 0.56 and 2.40 $\pm$ 0.07 with $\chi^2$/DOF = 4.05.

The validity of KNO scaling has been examined further by computing the normalized multiplicity moments, $C_q$.  

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Table 1. The mean multiplicity of projectile helium fragments, $C_q$ moments and the ratio ($n_n$)/D for different beams ($^{28}$Si and $^{32}$S) and different energies.

<table>
<thead>
<tr>
<th>Energy (A GeV)</th>
<th>$^{28}$Si</th>
<th>$^{28}$Si</th>
<th>$^{28}$Si</th>
<th>$^{32}$S</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>3.7</td>
<td>14.6</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>($n_n$)</td>
<td>1.83 $\pm$ 0.08</td>
<td>1.74 $\pm$ 0.08</td>
<td>1.73 $\pm$ 0.07</td>
<td></td>
</tr>
<tr>
<td>$C_2$</td>
<td>1.32 $\pm$ 0.06</td>
<td>1.31 $\pm$ 0.06</td>
<td>1.33 $\pm$ 0.05</td>
<td></td>
</tr>
<tr>
<td>$C_3$</td>
<td>2.21 $\pm$ 0.09</td>
<td>2.17 $\pm$ 0.09</td>
<td>2.18 $\pm$ 0.10</td>
<td></td>
</tr>
<tr>
<td>$C_4$</td>
<td>4.19 $\pm$ 0.18</td>
<td>4.28 $\pm$ 0.02</td>
<td>4.66 $\pm$ 0.18</td>
<td></td>
</tr>
<tr>
<td>$C_5$</td>
<td>8.98 $\pm$ 0.39</td>
<td>9.44 $\pm$ 0.42</td>
<td>10.98 $\pm$ 0.43</td>
<td></td>
</tr>
<tr>
<td>($n_n$)/D</td>
<td>1.79 $\pm$ 0.08</td>
<td>1.78 $\pm$ 0.08</td>
<td>1.75 $\pm$ 0.07</td>
<td></td>
</tr>
</tbody>
</table>

References

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\[ C_q = \langle n_q \rangle / \langle n \rangle^q, \quad (3) \]

where \( q = 2, 3, 4, \) and 5. The multiplicity scaling implies that \( C_q \) moments are energy independent since

\[ C_q = \langle Z^q \rangle = \int Z^q \Psi(Z) dZ. \quad (4) \]

In Table I, we have shown our results on the average multiplicities, \( C_q \) moments and \( \langle n_{a} \rangle / D \) of projectile helium (\( Z = 2 \)) fragments and have made a comparison with the values from \( ^{28}\text{Si–emulsion} \) interactions at 0.4 A GeV,\(^{14}\) 3.7 A GeV\(^{80}\) and \( ^{32}\text{S–emulsion} \) interactions at 200 A GeV\(^{89}\) respectively. Within experimental errors, \( C_q \) values do not reflect any significant change when energy increases from 0.4 to 200 A GeV. These observations once again confirm that \( C_q \) moments do not depend upon the mass and energy of a given projectile beam and have universal values. It is interesting to see that \( \langle n_{a} \rangle / D \) are also constant over a wide energy range (3.7 to 200 A GeV) for experiments with beams of nearly the same mass number. The second Muller moment\(^{60}\) is also calculated by using the following equation:

\[ f_{2} = (C_{2} - 1)\langle n_{a} \rangle^2 - \langle n_{a} \rangle, \quad (5) \]

and its value comes out to be 0.79. A non zero value of second Muller moment indicates a strong correlation exists among the projectile helium (\( Z = 2 \)) fragments.

4. Mean Multiplicities

In Table II, the mean multiplicity of shower particles \( \langle n_s \rangle \) and heavily ionizing particles \( \langle n_{h} \rangle \) in individual projectile helium fragments reaction channels are shown. A comparison of our present data on \( \langle n_{s} \rangle \) and \( \langle n_{h} \rangle \) at 14.6 A GeV energy has been made with the published \( ^{28}\text{Si data} \) at 3.7 A GeV.\(^{83}\) From Table II, one can observe that mean multiplicity of shower particles in individual projectile helium fragment channel as well as for all projectile helium fragment channel increases as the energy increases from 3.7 to 14.6 A GeV for same projectile (\( ^{28}\text{Si} \)) beam. While, one can observe that mean multiplicity of shower particles \( \langle n_{s} \rangle \) decreases as we move from 1He reaction channel to 4He reaction channel at 3.7 A GeV as well as at 14.6 A GeV energy, which shows that with decreasing the degree of centrality of the collision events, the number of produced particles (shower) reduces. Where as, the degree of target excitation represented by \( \langle n_{h} \rangle \) is correlated directly with the different projectile helium fragment channels. When more break up of projectile fragmentation (i.e., smaller the sum of \( Z = 2 \) PF\(_{i}\)) will occur, we observe more break up of target fragmentation. For example, in the case of one helium reaction channel, we have \( \langle n_{h} \rangle = 8.55 \pm 0.53 \) which is the largest among all helium reaction channel events. The multiplicity of projectile helium fragments \( \langle n_{a} \rangle \) can be regarded as a quantitative measure of the degree of projectile excitation. In Fig. 3, we have shown the correlation between the mean multiplicity of target fragments \( \langle \langle n_{h} \rangle \rangle \) and the multiplicity of projectile helium fragments. The target excitation \( \langle n_{h} \rangle \) is found to have a linear dependence on the projectile excitation \( \langle n_{a} \rangle \). The variation of \( \langle n_{h} \rangle \) as a function of \( n_{a} \) can be described by the following linear relation having the form \( \langle n_{h} \rangle = a + bn_{a} \), where \( a = 9.76 \pm 0.43 \) and \( b = -1.35 \pm 0.20 \) with \( \chi^2 = 1.05 \) per degree of freedom. So, we can see that the production of projectile helium fragments which are the decay products of projectile spectator has a strong correlation with the production of target fragments. For central collision events (i.e., \( n_{a} = 0 \)), the projectile and target completely overlap, the projectile spectator is zero and the excitation of target spectator is highest so the mean multiplicity of target fragments (i.e., \( \langle n_{h} \rangle \)) is the maximum. Whereas, in the quasi-central and peripheral collision events, the projectile spectator increases with the increase of the impact parameter, that is why the production probability of projectile helium fragments increases, but the target spectator decreases (due to low excitation of target) with the increase of impact parameter and therefore the mean multiplicity of target fragments decreases.

5. Angular Distribution of Projectile Helium Fragments

In Fig. 4(a), the projected angular \( \theta_{p} \) distribution in the plane of emulsion for all identified projectile helium fragments with charge \( Z = 2 \), has been shown. Here, we observe that 35% of the projected helium fragments are emitted with \( \theta_{p} \approx 0^{\circ} \) as a maxima of the distribution. The

![Table II. Mean multiplicities of shower and heavily-ionizing particles in different helium reaction channels from \( ^{28}\text{Si–emulsion} \) interactions at 3.7 A GeV\(^{80}\) and at 14.6 A GeV energy.](image-url)
projected angular distribution of emitted projectile helium fragments is mainly confined to a narrow forward cone of 1° with very few projectile helium fragments emitted at little larger angles up to 1.2°. Our data for projected angular distribution is fitted with a Gaussian distribution of the form \( N(\theta_p) = A \exp(-\theta_p^2/2\sigma^2) \) having standard deviation \( \sigma = 0.34 \pm 0.06° \). This permits a favourable comparison with \( \sigma = 0.56 \pm 0.03° \) observed in a similar experiment on the projectile helium fragmentation of 3.7 A GeV \(^{28}\)Si nuclei. From the projected angular measurements of projectile helium fragments, the space angular measurement \((\theta_s)\) can also be easily made and is shown in Fig. 4(b).

In Fig. 4(b), we observe a decreasing trend of the space angle distribution of projectile helium fragments with a peak around \( \approx 0.2° \). Here, we can see that the emission of projectile helium fragments is limited within a narrow forward cone of 2°. Most of these projectile helium fragments (\( \approx 85\% \)) are emitted with a space angle \( \theta \approx 0.5° \). There are very few projectile helium fragments which are emitted with a space angle \( \theta \geq 0.5° \). The observed value of \( \sigma \) is related to the width of the momentum distribution \( [\sigma(p)] \) in the projectile frame by the following relation,

\[
\sin[\sigma(\theta)] = \sigma(p) A_B \frac{p_B}{p_A F_A},
\]

where \( p_B \) is the beam momentum per nucleon. \( A_B \) and \( A_F \) are the mass numbers of the incident beam and projectile helium fragments, respectively. The width of momentum distribution \( \sigma(p) \) can be evaluated by using the formulation of Lepore and Riddell,\(^{17}\) in which the fragmentation of high energy nuclei was treated by using a quantum mechanical sudden approximation and obtained the following expression for \( \sigma(p) \)

\[
\sigma(p) = \left[ m_n (45A_B^{-1/3} - 25A_B^{-2/3}) A_F (A_B - A_F) \right]^{1/2},
\]

where \( m_n \) is the nucleon mass. According to the prescription of Feshbach and Huang,\(^{18}\) \( \sigma(p) \) is related to the Fermi momentum \( (p_F) \) of the nucleons within the incident beam.

Using the Goldhaber’s relation based on Feshbach and Huang’s work, \( \sigma(p) \) is given in terms of \( p_F \) by following relation:\(^{19}\)

\[
\sigma^2(p) = \frac{p_F^2}{5} \left[ \frac{A_F (A_B - A_F)}{A_B - 1} \right].
\]

Assuming that before disintegration, the excited non-overlapping part of the projectile comes to thermal equilibrium at temperature \( T \), the observed value of \( \sigma(p) \) can be related to the equilibrium temperature \( T \) by\(^{19}\)

\[
E_0 = kT = \frac{\sigma^2(p) A_B}{m_n A_F (A_B - A_F)}. \tag{9}
\]

The calculated values of \( \sigma(\theta_p) \), \( \sigma(p) \), Fermi momentum \( (p_F) \) and temperature for projectile helium fragments in nucleus–nucleus interactions at 14.6 A GeV are found to be 0.14°, 135.75 MeV/c, 164.84 MeV/c, and 6.05 MeV, respectively. In Fig. 5, the pseudo-rapidity distribution of projectile helium fragments emitted from the \(^{28}\)Si–emulsion interactions at 14.6 A GeV is shown. The pseudo-rapidity \( (\eta = -\ln \tan \theta/2) \) of each projectile helium fragment is calculated

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**Fig. 4.** Projected angle and space angle distribution of projectile helium fragments in minimum bias events \((n_b) \geq 0 \) events.

**Fig. 5.** Pseudo-rapidity \( (\eta_{lab}) \) distribution of the projectile helium fragments. Solid curve is a Gaussian fit to the projectile helium fragment data for minimum-bias events.
6. Transverse Momentum Distribution of Projectile Helium Fragments

The analysis of momentum distributions of projectile helium fragments can shed some light on the dynamics of the fragmentation process in nucleus–nucleus interactions, as they are major part of projectile fragments. The transverse momentum of individual projectile helium fragments in each interaction can be calculated indirectly from the space angle measurements of individual projectile fragments space angle by using the following expression:

\[ p_T = A_T p_0 \sin \theta, \]

where \( \theta \) is the space angle of the projectile helium fragments. \( A_T \) is the average mass number of the projectile helium fragments which comes to be 3.78 and \( p_0 \) is the incident projectile beam momentum per nucleon. In Fig. 6, we have shown the \( p_T \)-distribution of 861 projectile helium (\( Z = 2 \)) fragments emitted in 855 minimum-bias interaction events. The peak of the distribution is found near 200 MeV/c and the distribution has a long tail which extends up to a large value of 1150 MeV/c. Our data for \( p_T \) distribution is fitted with a Gaussian function having a mean value at 261.40 ± 7.50 MeV/c and standard deviation \( \sigma = 173.50 \pm 10.03 \) MeV/c. In inset, the \( p_T \) distribution of projectile helium fragments emitted with a momentum up to a value 550 MeV/c is shown. The distribution is fitted with a Gaussian function with \( \chi^2/\text{DOF} = 2.63 \) having the standard deviation \( \sigma = 145.00 \pm 6.00 \) MeV/c and the mean value of the distribution is \( \langle p_T \rangle = 205.90 \pm 7.50 \) MeV/c. One can observe that some of the projectile helium fragments are emitted with a value of transverse momentum greater than 550 MeV/c. The long tail of the distribution with high momentum >550 MeV/c corresponds to those alpha particles which (when emitted from the residual projectile nucleus) have definitely experienced a bounce off effect from a target nucleus.20 Analogous to the Coulomb deflection trajectory used for the measurement of impact parameter in low energy nuclear collisions, the measurement of bounce-off effect can be used for impact parameter measurements in relativistic high energy nucleus–nucleus collisions.21 However, the number of these high momentum helium fragments in an individual events with multiple helium events is not more than one.

Further, we can analyze the transverse momentum distribution of projectile helium fragments produced in nucleus–nucleus interactions at 14.6 A GeV by using two source emission model. According to the two source emission picture, the final-state transverse momentum \( (p_T) \) distribution of projectile helium fragments can be expressed as the sum of two Rayleigh distribution,22:

\[
f_{p_T}(p_T) = A_H \frac{p_T}{2 \sigma_H^2} \exp\left(-\frac{p_T^2}{2 \sigma_H^2}\right) \]

\[
+ A_L \frac{p_T}{2 \sigma_L^2} \exp\left(-\frac{p_T^2}{2 \sigma_L^2}\right),
\]

where \( \sigma_H \) and \( \sigma_L \) are the widths of transverse momentum distribution of projectile helium fragments emitted from the sources with high and low temperatures, respectively. \( A_H \) and \( A_L \) denote the contributions of the two sources. Two sources with high and low temperatures are also used for low energy reactions.23 In Fig. 6, the solid curve is the two Rayleigh distribution, which fits well with experimental data points and gives a good description of transverse momentum distribution of projectile helium fragments in nucleus–nucleus interactions at 14.6 A GeV.

7. Summary and Conclusions

From the extensive analysis of projectile helium \( (Z = 2) \) fragments in 28Si–emulsion interactions at 14.6 A GeV, we conclude that the angular and derived pseudorapidity and transverse momentum distributions of the projectile helium \( (Z = 2) \) fragments in 28Si–emulsion interactions at 14.6 A GeV are Gaussian shaped with widths \( \sigma \). The multiplicity distribution of projectile helium \( (Z = 2) \) fragments shows a reasonable agreement with a KNO-type scaling law and the normalized multiplicity moments \( C_q \) and \( \langle n_{0}\rangle/D \) are found to be independent of the mass and energy of the incident projectile beam within experimental errors. We also observe that the target fragment multiplicities \( \langle n_{f}\rangle \) decreases monotonically with increasing total charge of the emitted \( Z = 2 \) PF. Therefore we may conclude that the degree of excitation energy of the projectile and target nuclei are directly correlated and is independent of beam energy.

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