# Microsoft Excel Software Usage for Teaching Science and Engineering Curriculum* 

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#### Abstract

In this paper, our main objective is to present the use of Microsoft Software Excel 2007/2003 for teaching college and university level curriculum in science and engineering. In particular, we discuss two interesting and fascinating examples of interactive applications of Microsoft Excel targeted for undergraduate students in (1) computational physics and physics education, and in (2) computer and medical sciences. We demonstrate the tremendous amount of computational power of the most recent Microsoft Excel 2007 Software, which is employed here to perform simulations of a projectile (may be a missile) launched from an airplane so as to hit a target on the ground using the simplest assumption of no air resistance during the projectile motion in air and that of rolling of nine dice with six surfaces. For projectile motion, kinematic equations based on Newton's laws of uniform motion have been employed to simulate projectile trajectory. We are going to prove through a plot of vertical distance as a function time that the projectile motion undergoes a parabolic path. However, modification in the simulation equations of the projectile motion, which includes the effect of air resistance, change in gravitational force with altitude, curvature in the surface of Earth, Coriolis force due to Earth's spin motion about its axis of rotation and wind speed, will be discussed in very near future. We will also plot a graph of normalized total score versus the maximum score for rolling of nine dice like in casino games.


## 1. INTRODUCTION

During past one decade, the development and advancement in high speed micro-computers such as IBM and Mac based PCs including portable laptops have become versatile classroom tools to teach undergraduate science and engineering courses in colleges and universities. These very fast microcomputer machines employ several software systems such as Excel, Access, Word, PowerPoint, Groove, InfoPath, OneNote, Outlook, Publisher, FrontPage, and object oriented computing languages like C++, C\#, Visual Basic (VB), Java Script, SQL etc., which are developed and designed by the Microsoft Company [1], SUN Microsystems [2], IBM [3], AT \& T Inc. Research Labs [4]. The above mentioned software systems are frequently and extensively used for undergraduate, graduates teaching by instructors in educational institutes, scientific labs, private companies, businesses and banks all over the entire world and have become an integral part of modern life style [5].

In recent years, numerous American, European and Asian colleges and universities have already adopted internet technologies in undergraduate science and engineering curricula [6], and several international/national conferences have been held to enhance and share the knowledge gathered in such conferences with other educators and scientists [7]. With the advent of the Internet, it has become possible to interactively teach, in undergraduate and graduate classroom setting or during distance learning in virtual universities, very effectively
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the science and engineering curricula, which is one of the main motives of the present paper.
The paper is organized as follows: Section 2 describes how Microsoft Excel software system could be adapted for and taught to the college and university students. In Section 3, we explore the graphic capabilities of Microsoft Excel software. Section 4 is devoted to the development of the projectile motion problem in resistance free air. Section 5 deals with the theoretical formalism and its algorithm and Section 6 repots an actual simulation of projectile motion. Section 7 reports simulation of rolling of nine dice in a casino, and we also plot normalized total score versus the number of trials. This plot is similar to an Electrocardiograph (ECG) of an uneven heart-beat. Finally, concluding remarks of present study are made in Section 8.

## 2. ADAPTION AND TEACHING WITH MICROSOFT EXCEL SOFTWARE

Although there are numerous software systems available such as Mathematica, MatLab, Maple, Microsoft Studio .NET 2005, 2008 etc., which can be used for effective teaching tools, we preferred to employ Microsoft Excel software system for the present investigation. This is due to two main reasons: (i) Microsoft Excel is one of the software systems included in the Microsoft Office product, and therefore its user does have to shell out extra money to purchase it. In addition, almost all the New York State colleges/universities have Microsoft Office license, and thus instructors/students have an easy access to this software in their respective educational institutes, which is not true for rest of the software systems as mentioned above. (ii) Microsoft Excel has a user friendly Graphical User Interface (GUI), so that its beginner user can easily master its usage within a few hours [5].

From our personal experience of employing Microsoft Excel software system as a very useful teaching and research tool from the past more than fifteen years, it is our opinion that Excel could be adapted as a powerful teaching tool both by instructors and students to create simulations of several science, engineering, medical, business, arts and humanity etc. application principles. To learn and apply software system such as Microsoft Excel 2007 to the advanced science and engineering application principles, we advise that the interested instructors/students must be proficient in the basic knowledge of integral and differential calculus. The present paper is targeted for such like instructors/students who had prior, sound knowledge of integral and differential calculus, and it provides an in-depth exploration of the two classroom applications and their educational benefit. More specifically, both instructor and students could explore methods and strategies for applying Microsoft Excel to their mutual teaching/learning process, in order to create technology-enhanced learning experiences, to assess learning, and to facilitate collaboration and cooperative learning experiences. Through hands-on experiences with these two important applications (projectile motion in frictionless medium and rolling of nine dice) discussed in this paper, instructors/students would learn how to use Microsoft Excel in order to develop instructional tools and application principles that could be effectively employed in the classroom. We earnestly hope that current paper is an attempt to enhance educator's and student's skill who are very much interested to seek ways to expand their teaching/learning expertise by applying increased technology skills as a constructive means to greater and improved teaching/learning process.

## 3. GRAPH PLOTTING CAPABILITIES OF MICRSOFT EXCEL SOFTWARE

The major advantage of Excel 2007 is its ability to plot graphically any kind of data set in the form of a chart or graph. This versatile software system offers multiple options to present a given data set in a visual form, which is appealing to the eye and easy to evaluate. Further more, graphic representations of data make it much easier to present the useful information contained in a comparatively large amount of data-set collected concerning an experimental observation. Excel 2007 has a very powerful, built-in capability of graph plotting, which can be explored through something called an "Insert" Tab on the Excel 2007 GUI. Clicking on this "Insert" Tab, will take a user to several options of "Groups": our "Group" of current interest is the "Charts" Group, which could launch the "Charts" dialog box to choose a required chart type to be inserted depending on one's pertinent need. As shown in Fig.1, Excel 2007 can allow a user to a display graph in different forms (e.g., Column, Line, Pie, Scatter, Bar, Area, Other Charts etc.) in two- and three-dimensions. It is possible to add Titles and Axes to the graph, and change its color scheme. There is also an option of adding a Legend to the graph. Additionally, one has the choices of setting Text size, changing the Background Color, and loading and exporting graphs. For the present investigation, we will be using " $X Y$ (Scatter)" plot option in Sections 6 and 7 to plot the results of our simulation for the projectile motion and that of rolling of nine dice problems.


Fig.1: This figure is a graphing window in Microsoft Excel 2007 to insert a given kind of plot. It shows different types of graphs (shown in the LHS window) that can be plotted in Excel 2007 and it also gives a user an option of creating custom types of graphs. First column bar chart is shown ready for plotting. Scatter plots are in the last row on the RHS window.

## 4. DEVELOPMENT OF PROJECTILE MOTION PROBLEM

Many fascinating examples of projectile motion are: launching of a missile from an air plane on an enemy post, an artillery shell, a cannon ball, and hitting of a baseball with bat, a golf ball with golf club, shooting of an arrow with a bow etc. According to Aristotle's model, motion of a projectile like an arrow moving through air could be discussed as follows: Arrow moves horizontally with great speed and then drops to ground, which happens due to the force of bow applied to an arrow, and then due to air rushing in to fill vacuum behind arrow, causes (unnatural) motion with constant speed. Once, the force applied by the bow is completely exhausted, the arrow reverts to its natural state of rest on the ground. Newton later on modified the idea of natural rest state of motion in terms of his "First Law of Motion" in his famous book published in 1687, Principia Mathematica [8].

Some practical examples of projectile motion in physics curriculum are: launching of artillery shell, firing of a bullet from a gun, hitting a golf ball with a golf club, firing of a cannon ball with a cannon, launching of a cruise a missile on enemy target etc. As taught in freshman physics and engineering curriculum, projectile trajectory in air under the action of gravitational force with constant acceleration, $g\left(=9.8 \mathrm{~m} / \mathrm{s}^{2}\right)$, is a parabola. There are two types of particle models to study the projectile motion under uniform gravity: (i) analytic and (ii) dynamic [9]. An analytic model defines position of projectile as a function of time, while a dynamic model defines force functions and initial conditions for numerical solutions. Both models give qualitatively reasonable answers, but are quantitatively wrong since they miss many important physical effects, which can be simulated numerically, e.g., the air resistance being very complex and may depend on atmospheric pressure, temperature and air density etc. In addition, force may depend on shape of object (Magnus force due to object's spin motion), wind speed, variation of $g$ with altitude, curvature of the surface of Earth, Coriolis force etc [10].

## 5. THEORETICAL FORMALISM AND ALGORITHM OF PROJECTILE MOTION

To develop theoretical formalism and algorithm of the projectile motion in two-dimensional space, one needs to find position coordinates, $x(t)$ and $y(t)$ in addition to the projectile velocity $v(t)$ using Newton's equations of linear motion. To write down the equations in coupled first order form, Euler's method can be applied to each equation. We start with the basic definition of particle velocity in differential form along $x$ - and $y$-directions in two-dimensional space

$$
\begin{equation*}
v_{x}=\frac{d x}{d t}, \quad v_{y}=\frac{d y}{d t} . \tag{1}
\end{equation*}
$$

The projectile acceleration components are

$$
\begin{equation*}
a_{x}=\frac{d v_{x}}{d t}, \quad a_{y}=\frac{d v_{y}}{d t} . \tag{2}
\end{equation*}
$$

The force components along $x$ - and $y$-directions are

$$
\begin{equation*}
F_{x}\left(x, v_{x}, t\right)=m a_{x}, \quad F_{y}\left(y, v_{y}, t\right)=m a_{y} . \tag{3}
\end{equation*}
$$

Rearranging Eq. (1) and Eq. (2), and then integrating, we obtain

$$
\begin{array}{ll}
x=\int d x=\int v_{x} d t, & y=d y=\int v_{y} d t . \\
v_{X}=\int v_{x}=\int a_{x} d t, & v_{y}=\int v_{y}=\int a_{y} d t . \tag{5}
\end{array}
$$

Using constant acceleration as is the case for gravity $\left(g=-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)$, we can easily prove from Eq. (4) and Eq. (5) that

$$
\begin{align*}
& v_{x}=v_{0 x}+a_{x} t, \quad v_{y}=v_{0 y}+a_{y} t .  \tag{6}\\
& x=x_{0}+v_{0 x} t+\frac{1}{2} a_{x} t^{2}, \quad y=y_{0}+v_{0 y} t+\frac{1}{2} a_{y} t^{2} . \tag{7}
\end{align*}
$$

where $x_{0}$ and $y_{0}$ are initial position coordinates and $v_{0 x}$ and $v_{0 y}$ are initial components of velocity of projectile along $x$ - and $y$-directions, respectively. Eq. (6) and Eq. (7) are known as kinematic equations of the projectile motion in two dimensional space and we are going to employ these equations to simulate the projectile trajectory under action of gravity with the simplest assumption of no air resistance. Applying the boundary conditions for the present problem (i.e. $a_{x}=0, a_{y}=g=-9.80 \mathrm{~m} / \mathrm{s}^{2}, v_{y}=V, v_{0 y}=V_{0}, y=H$, and $y_{0}=H_{0}$ ), Eq. (6) and Eq. (7) can be written in the following form along $y$-axis as

$$
\begin{equation*}
V=V_{0}+g t, \quad H^{\prime}=H_{0}+V_{0} t+0.5 g t^{2} \tag{8}
\end{equation*}
$$

These equations will be used in Section 6 to simulate the projectile motion to simulate its exact velocity $V$ and exact height $H^{\prime}$ at a given instant of time.

## 6. INTERACTIVE SIMULATION OF PROJECTILE MOTION UNDER FREE FALL

To simulate the projectile motion in two-dimensional space, we will use Microsoft Excel 2007 spreadsheet software system for the current problem. Excel spreadsheet is computerized equivalent of a ledger that was used in banks to maintain account information of customers. Spreadsheet is composed of 256 columns and 65536 rows [5]. In an Excel interface [5], each row is assigned numeric heading (such as 1, 2, 3, 4,......) and each column is given alphabetic heading, e.g., A, B, C, D, and so on. Each column proceeds along vertical direction ( $y$-axis) and each row is displayed along horizontal direction ( $x$-axis). Intersection of each column with a given row is called a cell. Each cell can contain data in the form of digits/text or a combination of both. Each cell has a unique cell reference, which is the address of that cell. Cell reference is designated by column alphabet followed by row number to which the cell belongs. Cell reference may be absolute, relative or a combination of both (called mixed reference), depending on the kind of cell formula to be designed to perform a given simulation or type of built in function to be used in Excel. For the present simulation, we do not employ the concept of absolute and mixed cell references. One may use vertical or horizontal arrow keys to navigate through a worksheet. Hidden behind each cell is something known as the cell formula, which can be used to do simulations in terms of some
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built-in functions in Excel using the Function Wizard or one may design his/her own cell formula using particular cell reference or several references along with usual common mathematical operators: exponentiation, ^; division, /; multiplication, *; subtraction -; addition, + . Eq. (8) has been used to simulate projectile motion with the help of Microsoft software Excel 2007 [5]. The cell formula in Excel always starts with an equals sign (=), and thus the corresponding cell formulas of Eq. (8) for simulation of exact velocity and height should be typed in Excel spreadsheet as

$$
\begin{align*}
& V=V_{0}+g^{*} \mathrm{~A} 2  \tag{9}\\
& H^{\prime}=H_{0}+V_{0}^{*} \mathrm{~A} 2+0.5^{*} g^{*} \mathrm{~A} 2^{\wedge} 2 \tag{10}
\end{align*}
$$

where $V_{0}=0 \mathrm{~m} / \mathrm{s}$ and $H_{0}=100 \mathrm{~m}$ is the value of initial velocity and height of the projectile in $y$-direction, and A2 $=d t=0.0125 \mathrm{~s}$ represents the relative cell reference for an increment of time interval, $d t$, which is memorized in Excel by defining something known as "Defined Name" [5] and its value may exist in a different cell, whose cell reference could be used in Eq. (9) and Eq. (10) for the current simulation work. The same outcome can be achieved by putting the value of $d t$ in a given cell (A2), but without using the idea of "Defined Name" provision. The computed height $H$ is always a little less than that of the exact height $H^{\prime}$. For $93 \%$ of the simulated data points, the magnitude of percent error between simulated height and actual eight is $<4.0 \%$, which indicates that the accuracy in computed values of projectile height is pretty good, which further proves that the chosen time interval $d t=0.0125 \mathrm{~s}$ almost satisfies the necessary and sufficient condition of differential calculus that in the limit of infinitesimal time interval, $\Delta t \rightarrow 0$ for the projectile motion.

Negative value of instantaneous velocity $V$ indicates that the projectile is moving along the downward direction, i.e., along negative direction of $y$-axis. In a total duration of 0.4875 s , the projectile just moved a little more that 1.0 m along negative $y$-axis. To complete a journey of total vertical distance of 100 m , the projectile took almost 5.0 s . In fact, to cover a time interval of $\approx 5.0 \mathrm{~s}$, total number of simulations is calculated to be $\approx 400$.

The computed height $H$ for the projectile motion is obtained by transforming E. (1) and Eq. (2) into change equations as follows:

$$
\begin{equation*}
v_{y}=\frac{\Delta y}{\Delta t}, \quad a_{y}=\frac{\Delta v_{y}}{\Delta t} \tag{11}
\end{equation*}
$$

Let us convert Eq. (11) so as to make them suitable for Excel:

$$
\begin{equation*}
V=V_{0}+g \Delta t, \quad H=H_{0}+V \Delta t . \tag{12}
\end{equation*}
$$

These equations are used in to simulate the projectile motion for the computed height, $H$. For $93 \%$ of the simulated data points, the magnitude of percent error between computed height and actual height is $<4.0 \%$, which indicates that the accuracy in computed values of projectile height is pretty good.

The magnitude of horizontal range of projectile during its total time of flight $t=4.775$, assuming airplane is moving at a constant speed of $V_{\text {airplane }}=500$ miles/hour along x-axis,
can be obtained from kinematic equation Eq. (7) by using the initial boundary conditions, i.e., $\mathrm{x}=R, a_{\mathrm{x}}=0, x_{0}=0$ and $v_{0 \mathrm{x}}=V_{\text {airplane }}$ :

$$
\begin{equation*}
\text { Horizontal Range, } R=t V_{\text {airplane }}=1067 \mathrm{~m} \tag{13}
\end{equation*}
$$

The horizontal range, $R$ is the distance where the projectile will hit a target on the ground. In the present problem, $R=1.07 \mathrm{~km}$, which can be increased either by increasing the airplane speed with respect to ground or by imparting some initial thrust to the projectile at launch time or by a combination of both.

We now plot the projectile simulated height $H$ as a function of its time $t$ in Fig. 2 for two different altitudes: (a) 100 m , blue curve and (b) 90 m , burgundy curve. One can easily perceive that the graph is a parabola as expected from the theory. The second order equation for the best parabolic fit through the two simulated data sets is: $y=4.9 x^{2}-0.0612 x$ $+c$, as expected from formulation in Section 5. Here, $c$ stands for the intercept of the respective curve at the launch point of projectile. In this parabolic equation, variable $x=t$ is the time of flight of the projectile and the intercept of the best fit curve with $y$-axis is: $c=100$ $\mathrm{m}, 90 \mathrm{~m}$, which is the respective initial height at which the projectile was launched from the airplane at time $x=t=0$. In Fig. 3, we present two slider bars that are used to perform simulations with different initial velocity $V_{0}$ of the projectile and at a different initial height $H_{0}$ of the airplane. Here, slide bar 1 represents the instantaneous initial height of the projectile, whereas slide bar 2 shows the initial velocity of the projectile at launch time. The initial height, $H_{0}$ and initial velocity, $V_{0}$ of the projectile can be increased or decreased by clicking on right or left hand side arrow existing on each end of the slide bar.


Fig. 2: A plot of computed projectile height, $H(\mathrm{~m})$ as a function of time, $t(\mathrm{~s})$ : (a) $H=100 \mathrm{~m}$, is for curve shown in blue color, and (b) $H=90 \mathrm{~m}$, belongs to curve shown in burgundy color. The incept $c=100 \mathrm{~m}$ stands for the bue curve in Fig. 2(a). The intercept value for the burgundy curve is, $c=90 \mathrm{~m}$, which is discussed in the text for the best-fit curve through the second simulated data set.


Fig. 3: Slide bar 1 is for changing initial value of the projectile height $H_{0}$ and slide bar 2 is for varying its initial velocity $V_{0}$.

## 7. INTERACTIVE SIMULATION OF ROLLING OF NINE DICE

To simulate the rolling process of nine dice in a casino game, once again, we employ the latest version of Microsoft Excel 2007 software. To do the simulations, we use a built-in pseudo number generating function called RAND( ), which can generate all kinds of fractional numbers between 0 and 1. As none of the faces of each dice has marked with zero a dot, one is bound to include this fact while generating the random numbers with the generating function RAND(). Cell formula to create non-zero random numbers for the rolling of nine dice should also include a factor of 6, which is multiplied with the pseudo number generating function RAND( ) to take into account the fact of six faces of each dice, and a factor of unity is added to it to get rid of zero value generated random numbers. The random numbers thus generated for nine rolling dice could be used to arrive at some very interesting conclusion. Now, we determine the sum total of scores obtained for all the nine dice in one trial and normalize it to the maximum score of a data-set of 200 simulated values and repeat this procedure for the entire values of simulated data. If one double clicks any cell of generated data in Microsoft Excel 2007 spreadsheet, and then hits the ENTER key on the keyboard, all simulated random numbers for nine dice will change instantaneously and consequently, the total score in a single row normalized with the maximum score of entire data values, will also change. In Fig. 4, we display a graph of this normalized total score as a function of number of trials. This graph has several peaks and valleys and it looks like the replica of an Electrocardiograph (ECG), which is obtained for a patient with some defect in the heart causing an irregular heart-beat. The interactive plot of Fig. 4 has in general, one or two peaks with a maximum value equals unity, and the remaining peaks always have values less than unity. The location of the maximum peak values and the nature of the plot changes with each new simulation, showing pretty interesting application of Excel 2007 for computer science and medical undergraduates.

## 8. CONCLUDING REMARKS

In the first example, we have presented for the undergraduate classroom setting the simulations of projectile motion falling freely from certain height under the action of gravity with no air resistance. Microsoft Software Excel 2007 [1, 5] has been used for the present simulations. Exactly the same simulations can be performed with Microsoft Excel 2003. Cell formulas for the kinematic equations of projectile motion have been developed. Slide bars have been used to impart a change in the initial value of projectile velocity and its height, so as to investigate the simulations under different initial boundary conditions. The projectile trajectory has been determined to be a parabola, which has been displayed through a plot of vertical distance $H$ as a function of time of flight $t$ of the projectile. For $93 \%$ of the simulated data points, $<4.0 \%$ difference between the simulated computed $H$ and actual $H^{\prime}$ heights has been found, which indicates that the necessary and sufficient condition of differential calculus under the limit of an infinitesimal change in time interval that tends to zero, i.e., $\Delta t \rightarrow 0$, has
been almost satisfied. We hope that the current simulations of projectile motion will be quite useful for the undergraduate students of computer science, computational physics and physics education to understand the basic physical concept of kinematics of a projectile falling freely under the action of gravity. Extension of the current simulations would be done in very near future to include the effect of wind speed, variation of acceleration due to gravity, $g$ with altitude, spherical curvature in the surface of Earth, Coriolis force introduced by Earth's spin motion about its rotational axis etc.


Fig. 4: A plot of ratio of total score in one row to the maximum score among all scores in column ten of Table 2 as a function of number of trials.

In another interesting application, we have done simulations for nine rolling dice in a casino game. A plot of total score in one trial divided by the maximum score as a function of the number of trials is displayed in Fig. 4, which looks like an Electrocardiograph (ECG) of a patient with an irregular heart-beat. This simulation is pretty exciting and interesting for the medical students. Also computer science students will learn here how to do random number simulations for rolling of nine dice in a casino game.

Further, we may emphasize that the present paper has quite important implications both in physics as well as in computer science and medical science curriculum: (i) In physics the students learn how to employ software system such as Excel 2007/2003 [1, 5] to simulate the basic concept of projectile motion under the action of constant gravitational acceleration with no air resistance, whereas in computer science, they visualize the real time application of this fundamental concept of physics in a virtual laboratory. In addition, medical students can have an idea of irregular heart-beat of a patient suffering from heart attack or stroke, which has been proven with the help of a plot of normalized total score as a function of the number of trials.
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In conclusion, we strongly believe that our efforts are very beneficial for college/university students of physics, engineering, computer science and medical sciences. Nowadays, almost all students have laptops with Microsoft Office software installed. Our Excel2007/2003 programs introduce them to the most basic problems in deterministic mechanics (projectile motion) and probability and statistics (throws of dice) without requiring any additional software or programming knowledge and experience. Moreover, our current use of interactive graphics encourages hands-on learning similar to performing an experiment in the laboratory. The best form of learning is by doing, which we hope would be appreciated by the college and university students.

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