

Objectives:

- To become familiar with the term **probability** and the idea that it is a random process based on an expected relative frequency of “successes” per events.
- To understand the use of **simulations** to approximate a probability by obtaining an **empirical Estimate**.
- To become familiar with use of **sample spaces** to represent the equally likely outcomes of an event.
- To understand the calculations of **Expected values** for an event to obtain a value for what would happen in the **long-run** or on **average** when performing a certain event.
- To understand the role of sample size on estimating probabilities using simulations and samples.

Probability: The long-run proportion of number “successes” to number of events for a **random** event if the event were to be performed numerous times.

Relative Frequency: The number of times a success occurs per number of events performed.

Simulation: A process that tries to mimic the numerical results of a long-run event. Often actually performing an event is either impossible or cost prohibitive yet we still need to calculate a probability of “success”. This may be done by computer, random number tables, shuffling and dealing cards, etc.

Empirical Estimate: the result obtained from a simulation to estimate a probability. An Empirical result is one derived from performing an event numerous times and finding the proportion from it.

Theoretical Estimate: A result obtained through mathematical calculation. A theoretical probability is calculated completely on paper with no need to perform any trials.

Sample Space: An exhaustive list of equally likely outcomes of a random event. Allows the calculation of a theoretical probability based on the number of possible successes divided by the total number of possible outcomes.

Expected value: The long-run average result of a process. For example if we have a game that involves rolling dice, the possible outcomes are 1 thru 6, costing you \$2.00 to play. The payoffs are listed in the following table:

Result	1	2	3	4	5	6
Payoff	\$0	\$0	\$1	\$1	\$2	\$5
probability	.167	.167	.167	.167	.167	.167

So the average payoff would be calculated by multiplying the probability by the payoff and adding these results together:

$$E(x) = \sum_{i=1}^n f(i)P(i)$$

$$E(x) = \text{Payoff}(1) \times \text{Prob}(1) + \text{Payoff}(2) \times \text{Prob}(2) + \text{Payoff}(3) \times \text{Prob}(3) + \text{Payoff}(4) \times \text{Prob}(4) + \text{Payoff}(5) \times \text{Prob}(5) + \text{Payoff}(6) \times \text{Prob}(6)$$

$$E(x) = (\$0 \times .167) + (\$0 \times .167) + (\$1 \times .167) + (\$1 \times .167) + (\$2 \times .167) + (\$5 \times .167)$$

$$E(x) = \$1.50$$

But you paid \$2.00 to play the game, so if the expected winnings is \$1.50 over the long run you end up losing \$0.50 per game if you were to play this game over and over.