Objectives:

1. To become familiar with the term ***probability*** and the idea that it is a random process based on an expected relative frequency of “successes” per events.
2. To understand the use of ***simulations*** to approximate a probability by obtaining an ***empirical Estimate***.
3. To become familiar with use of ***sample spaces*** to represent the equally likely outcomes of an event.
4. To understand the calculations of ***Expected*** ***values*** for an event to obtain a value for what would happen in the ***long-run*** or on ***average*** when performing a certain event.
5. To understand the role of sample size on estimating probabilities using simulations and samples.

***Probability:*** The long-run proportion of number “successes” to number of events for a ***random*** event if the event were to be performed numerous times.

***Relative Frequency:*** The number of times a success occurs per number of events performed.

***Simulation:*** A process that tries to mimic the numerical results of a long-run event. Often actually performing an event is either impossible or cost prohibitive yet we still need to calculate a probability of “success”. This may be done by computer, random number tables, shuffling and dealing cards, etc.

***Empirical Estimate:*** the result obtained from a simulation to estimate a probability. An Empirical result is one derived from performing an event numerous times and finding the proportion from it.

***Theoretical Estimate:*** A result obtained through mathematical calculation. A theoretical probability is calculated completely on paper with no need to perform any trials.

***Sample Space:*** An exhaustive list of equally likely outcomes of a random event. Allows the calculation of a theoretical probability based on the number of possible successes divided by the total number of possible outcomes.

***Expected value:*** The long-run average result of a process. For example if we have a game that involves rolling dice, the possible outcomes are 1 thru 6, costing you $2.00 to play. The payoffs are listed in the following table:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Result | 1 | 2 | 3 | 4 | 5 | 6 |
| Payoff | $0 | $0 | $1 | $1 | $2 | $5 |
| probability | .167 | .167 | .167 | .167 | .167 | .167 |

So the average payoff would be calculated by multiplying the probability by the payoff and adding these results together:

$$E\left(x\right)=\sum\_{i=1}^{n}f\left(i\right)P(i)$$

E(x) = Payoff(1)xProb (1) + Payoff(2)xProb (2) + Payoff(3)xProb(3) + Payoff(4)xProb (4) + Payoff(5)xProb (5)+ Payoff(6)xProb (6)

E(x) = ($0 X .167) + ($0 X .167) + ($1 X .167) + ($1 X .167) + ($2 X .167) + ($5 X .167)

E(x) = $1.50

But you paid $2.00 to play the game, so if the expected winnings is $1.50 over the long run you end up losing $0.50 per game if you were to play this game over and over.