

## CSIT 241 Quiz 2 Spring 2002

Name: .....

**Instructions:** Do all of the following and show your work.

1. (2 points) Prove or disprove

If  $n$  is an odd prime number, then  $n^2 + 4$  is prime.

**Solution:** False. Here is a counterexample. Take  $n = 11$ . 11 is prime and odd, but  $11^2 + 4 = 125$  which is not prime.

2. (2 points) Prove by contradiction

If  $x$  is irrational, then  $\sqrt{x}$  is irrational.

**Solution:** True. Proof is by contrapositive. To do that assume  $\sqrt{x}$  is rational and prove  $x$  is rational. So, assume  $\sqrt{x}$  is rational. This means there exist two integers  $p$  and  $q$  with  $q \neq 0$  such that  $\sqrt{x} = \frac{p}{q}$ . Now square both sides to get  $x = \frac{p^2}{q^2}$ . Notice that both  $p^2$  and  $q^2$  are integers because  $p$  and  $q$  are integers. Notice also that  $q^2 \neq 0$ , because  $q \neq 0$ . Thus,  $x$  is rational.

3. (4 points)

Let  $A = \{c, d\}$  and  $B = \{d, e\}$ . Find  $A \times B$  and  $A \oplus B$ .

**Solution:**

$$A \times B = \{(c, d), (c, e), (d, d), (d, e)\}.$$

$$A \oplus B = A \cup B - A \cap B = \{c, e\}.$$

4. (2 points)

Let  $h_0 = 1$ ,  $h_1 = 2$ , and  $h_n = h_{n-1} + 3h_{n-2}$ ,  $\forall n \geq 2$ . Find  $h_3$ .

**Solution:**  $h_3 = h_2 + 3h_1 = h_2 + 6$ . Thus, we need to find  $h_2$ . But,  $h_2 = h_1 + 3h_0 = 5$ . Therefore,  $h_3 = 11$ .